

Relations among Security Metrics for Template Protection Algorithms

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Abstract

Many biometric template protection algorithms have been proposed mainly in two approaches: biometric feature transformation and biometric cryptosystem. Security evaluation of the proposed algorithms are often conducted in various inconsistent manner. Thus, it is strongly demanded to establish the common evaluation metrics for easier comparison among many algorithms. Simoens et al.[11] and Nagar et al.[1] proposed good metrics covering nearly all aspect of requirements expected for biometric template protection algorithms. One drawback of the two papers is that they are biased to experimental evaluation of security of biometric template protection algorithms. Therefore, it was still difficult mainly for algorithms in biometric cryptosystem to prove their security according to the proposed metrics. This paper will give a formal definitions for security metrics proposed by Simoens et al.[11] and Nagar et al.[1] so that it can be used for the evaluation of both of the two approaches. Further, this paper will discuss the relations among several notions of security metrics.

1 Introduction

One of the main issues in biometric authentication systems is to protect a biometric template database from compromise. Biometric information is so unique to each user and unchangeable during his or her lifetime. Once biometric template is leaked together with his or her identity, the person will face a severe risk of identity theft. Widely-used template protection systems for biometric authentication systems are tamper-proof hardware-based systems, where biometric template is stored in an ordinary storage as an encrypted form and decrypted only within a tamper-proof hardware when matching is required. In these systems, even if the database is compromised, biometric information never made public. However, the drawback of this conventional approach was the requirement of tamper-proof hardware, as it increases the deployment cost especially in high volume matching is required. To overcome this drawback, software-based template protection techniques are proposed recently in many literature[.]. Software-based template protection schemes are categorized into 2 approaches[12], feature transformation approach and biometric cryptosystems. Both of them introduces a user-specific key to transform a biometric template into a protected template.

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1.1 Feature Transformation approach

Feature transformation approach is first proposed in a paper written by Ratha, Connell and Bolle[14] as *Cancelable biometrics*. In feature transformation approach, a randomness or key is introduced as a transformation parameter, and each original biometric feature is transformed into a deformed biometric feature. Main advantage of this approach is that it can take benefits from utilizing well-studied high performance algorithms. Thus, the challenge in this approach is to design a transformation function satisfies both (1) that closeness in original biometric feature space should preserve in the transformed feature space and (2) that it is hard to recover the original biometric feature from the transformed feature. On the contrary that feature transformation approach can enjoy the benefit of high-performance algorithms, schemes in this approach tends to have difficulties in theoretical analysis of protection performance such as irreversibility and unlinkability discussed later. Thus, many papers give experimental evidence for security analysis.

Ratha et al. [14] introduced the notion of *Cancelable biometrics* and proposed several schemes for fingerprint template protection[13]. Their approach is to displace fingerprint minutiae at different locations according to a irreversible locally smooth transformation. That is, a small change in a minutiae position before transformation leads to a small change in the minutiae position after transformation, but small correlation in minutia positions before and after transformation. Ratha et al.[13] evaluated *Accuracy* (Section 3.1) for the recognition performance and *Irreversibility*(Section 4.1) for their schemes. They roughly estimated the complexity of irreversibility by the length of its binary representation.

Teoh et al.'s BioHash[8] and its subsequent papers[4, 16, 17] proposed distance-preserving transformations for biometric feature vectors multiplied with an randomized orthogonal transformation matrix. The randomized orthogonal matrix works as a user-specific key, it introduces a low false accept rate. Irreversibility of BioHash is analyzed in [8] and [18]. In [8], irreversibility is discussed based on evidences from *recognition performance* (Section 3) metrics such as *accuracy* (Section 3.1), *biometric performance* (Section 3.2) and *diversity* (Section 3.3). As argued later, for example, in the real world, a fingerprint left on a glass may be abused by a malicious user, then *Diversity* seems to give the complexity of an adversary to find the correct key. However, this discussion only covers a weak adversary whose attacking strategy is specific. A stronger adversary may take other strategies such as finding the correct key by directly inverting the transformation function utilizing the stolen fingerprint, etc. Likewise, those recognition performance metrics are not suitable for the evaluation of protection performance. In [18], irreversibility is discussed theoretically and experimentally. Their experimental analysis is similar to [8]. In their theoretical analysis, irreversibility is defined as the complexity of finding an exact original biometric feature vector from a transformed template and its corresponding key. BioHash is a lossy function, hence it satisfies their notion of irreversibility with some security parameter. However, in the real situation, the adversary usually does not have to find an exact original biometric feature, but enough to find an biometric feature which can be accepted by the biometric authentication system. The latter is trivially easy, given a transformed template and its corresponding key, randomly chosen biometric features will be accepted with probability FAR. Thus, more realistic notion of irreversibility is required.

1.2 Biometric cryptosystem

Biometric cryptosystem refers to a series of research motivated by fuzzy commitment and fuzzy vault proposed by Juels and Watenburg[10] and Juels and Sudan[9] respectively. Instead of applying

sophisticated feature extraction and matching algorithms, they abstracted the metric space of biometrics matching as a hamming distance or a set difference respectively, and make use of error-correcting codes to check if the distance of two biometric features are within a correctable range. Dodis, Reyzin and Smith[5] generalized them to secure sketch covering any *transitive* metric space, that is, a metric space \mathcal{M} has a family of permutations $\pi \in \Pi$ such that Π is distance preserving: $d(a, b) = d(\pi(a), \pi(b))$ and for any two elements $a, b \in \mathcal{M}$ there exists $\pi_i \in \Pi$: $\pi_i(a) = b$.

None of them conducted experimental analysis both on *recognition performance* (Section 3) and *protection performance* (Section 4). Rather, *irreversibility* (Section 4.1) for their *un-keyed* schemes are theoretically analyzed. They demonstrated that fuzzy schemes have strong *irreversibility* in a practical parameter setting, but introduced impractical assumptions. As shown in this paper, any *un-keyed* schemes cannot satisfy *irreversibility* in a practical setting for a biometrics application (see Theorem 2 in this paper). Those impractical assumptions are considered essential in the analysis. Namely, Juels and Watenburg[10] assumes uniform distribution on biometric features, and Juels and Sudan[9] does not assume uniform distribution on elements in a set whereas assumes elements in a set are chosen independently. Dodis, Reyzin and Smith[5] evaluated *irreversibility* of secure sketch and fuzzy extractor with a general distribution on biometric feature, hence falls to insecure with a practical parameter setting for biometrics applications.

Sutcu, Li and Memon[15] applied a secure sketch[5] to a face recognition system, and measured *biometric performance* and estimated a lower-bound of *irreversibility*. They reported degradation of recognition performance introduced by secure sketch was negligible, but the lower-bound of complexity to break *irreversibility* was barely 20 bits. Arakala, Jeffers and Horadam[2] and Chang and Roy[3] applied to fingerprint recognition system and reported similar results.

1.3 Related Security Metrics

As we have seen until now, there are two separate line of research, and there exists a gap in the way of evaluation of *recognition performance* and *protection performance* between feature transformation approach and biometric cryptosystem. Thus, relations of security statements were ambiguous, and it was not easy to compare the security of proposed schemes. Recently, there are attempts to try to unify the evaluation methods and give metrics applicable to all biometric template protection schemes.

Nagar, Nandakumar and Jain[12] proposed such security metrics. Their security metrics consists of six items: FAR_{UK} , FAR_{KK} , IRIS , IRID , CMR_{T} and CMR_{O} . The first two items exactly correspond to our proposal, *accuracy* and *biometric performance*. IRIS , the Intrusion Rate due to Inversion for the Same biometric system, and IRID , the Intrusion Rate due to Inversion for a Different biometric system, are related to our metric of ϵ - $\{PI, AD\}$ -pseudo-authorized leakage *irreversibility* in Definition 4. Our metric gives the upper-bound of the intrusion probability for all probabilistic polynomial-time inverters, whereas IRIS and IRID give the intrusion probability for the best possible inverter. IRIS and IRID can be evaluated experimentally, hence suitable metrics for algorithms in the feature transformation approach. However, IRIS and IRID should be considered that it gives the lower assurance in *irreversibility*, as far as there is no evidence that the best possible inverter used in the evaluation is the best of all probabilistic polynomial-time inverters. Similarly, CMR_{T} , the Cross Match Rates in the Transformed feature domain, and CMR_{O} , the Cross Match Rates in the Original feature domain, are related to our *diversity* and ϵ - $\{PI, AD\}$ -*unlinkability*, respectively in Definition 5.

Simoens, Yang, Zhou, Beato, Busch, Newton and Preneel[11] proposed nearly all aspect of requirements normally expected to template protection algorithms, namely from technical performance such as recognition accuracy, throughput and storage requirement, protection performance through operational performance. Based on their proposal, this paper focuses on the formal definitions of the recognition performance and the protection performance for precise discussions. For recognition performance, their *accuracy*[11] and *diversity*[11] exactly corresponds to our *biometric performance* and *diversity*. Further, we introduced another *accuracy* which corresponds to FAR_{UK} in Nagar et al.[12] to demonstrate the performance advantage of two-factor template protection algorithms. For protection performance, their *irreversibility*[11] and *unlinkability*[11] exactly corresponds to ours. *Irreversibility*[11] is further divided into *full-leakage irreversibility*, *authorized-leakage irreversibility* and *pseudo-authorized-leakage irreversibility* depending on the differences of goals for adversary. These three notions of *irreversibility* is formally defined and discussed their relations in Section 4.1. *Unlinkability*[11] is defined as the false cross match rate (*FCMR*) and the false non-cross match rate (*FNCMR*). These rate is measured as the performance of a *cross-comparator*. Similarly, if one could give an upper-bound of *FCMR* and *FNCMR* for all probabilistic polynomial-time *cross-comparator*, then *unlinkability* can be theoretically evaluated with the high assurance level. On the other hand, if these rates are given experimentally for the best possible *cross-comparator*, *unlinkability* is evaluated with lower assurance level. These are discussed in more detail in Section 4.2.

2 Preliminaries

In this section, we will explicitly formulate *biometric template protection (BTP) algorithms*. In this paper, we discuss BTP algorithms utilizing a common modality and a common feature extraction algorithm. Namely we do not discuss BTP algorithms using multi-biometrics.

Let \mathcal{U} be a finite set consisting of all users who have biometric characteristics utilized in BTP algorithms. Assume that each user $u \in \mathcal{U}$ has his/her own biometric characteristic b_u and therefore, in the following, we identify u with b_u and use the notation u instead of b_u , namely, the set \mathcal{U} can be regarded as a set consisting of all individuals' biometric characteristics. A biometric recognition system captures biometric samples from biometric characteristics presented to the sensor of the system, extracts biometric features from biometric samples, and verifies or identifies users by using their biometric features. We assume that each user's biometric features are represented as a digital element $x \in \mathcal{M}$ of a finite set \mathcal{M} . We call x a *feature element* of u . Since two feature elements generated from u are rarely identical, we let X_u denote a random variable on \mathcal{M} representing noisy variations of feature elements of u , namely $P(X_u = x)$ is the probability that a biometric sample of captured from u will be represented as x . Let \mathbf{R} be the set of all real numbers and let $d : \mathcal{M} \times \mathcal{M} \rightarrow \mathbf{R}$ be a *semimetric function* on \mathcal{M} , namely the real-valued function d satisfies the following three conditions:

- (i) $d(x, y) \geq 0$
- (ii) $d(x, y) = 0$ if and only if $x = y$
- (iii) $d(x, y) = d(y, x)$

for all $x, y \in \mathcal{M}$. Then \mathcal{M} is called a *semimetric space* associated with d . For any $x \in \mathcal{M}$, $\mathcal{M}_\tau(x) = \{x' \mid d(x, x') \leq \tau\}$ is called the τ -neighborhood of x . Let f be an algorithm (or a

function) on \mathcal{M} whose input $x \in \mathcal{M}$ is chosen according to a random variable X . Let $f(X)$ denote a random variable induced on the image of f . For any set T , the notation $t \stackrel{\$}{\leftarrow} T$ denotes that t is chosen from the set T uniformly at random. For any random variable X on a set \mathcal{M} , the notation $x \leftarrow X$ denotes that x is chosen according to X . For any function f on the set \mathcal{M} , the notation $\mathbb{E}_{x \leftarrow X} f(x)$ denotes the expected value of f under the condition that x is chosen according to the random variable X , namely

$$\mathbb{E}_{x \leftarrow X} f(x) = \sum_{x \in \mathcal{M}} \Pr[X = x] f(x) .$$

In particular,

$$\begin{aligned} \mathbb{E}_{x \leftarrow X} \Pr[\text{an event of } x] &= \sum_{x \in \mathcal{M}} \Pr[X = x] \Pr[\text{an event of } x \mid X = x] \\ &= \sum_{x \in \mathcal{M}} \Pr[X = x, \text{ an event of } x] . \end{aligned}$$

Traditional biometric comparison algorithms are assumed to utilize an ordinary comparison method which, for an enrolled feature element and a freshly extracted feature element x' during verification, decides *match* if $d(x, x') \leq \tau$, and otherwise *non-match* by using a decision threshold τ . Then, the *false non-match rate* $FNMR_{d \leq \tau}$ and the *false match rate* $FMR_{d \leq \tau}$ are formulated as follows:

$$\begin{aligned} FNMR_{d \leq \tau} &= \mathbb{E}_{\substack{u \stackrel{\$}{\leftarrow} \mathcal{U} \\ x, x' \stackrel{\$}{\leftarrow} X_u}} \Pr[d(x, x') > \tau] \\ FMR_{d \leq \tau} &= \mathbb{E}_{\substack{(u, v) \stackrel{\$}{\leftarrow} (\mathcal{U} \times \mathcal{U})^{\text{diff}} \\ x \stackrel{\$}{\leftarrow} X_u, y \stackrel{\$}{\leftarrow} X_v}} \Pr[d(x, y) \leq \tau] . \end{aligned} \tag{1}$$

where $(\mathcal{U} \times \mathcal{U})^{\text{diff}} = \{(u, v) \in \mathcal{U} \times \mathcal{U} \mid u \neq v\}$ and $\#\mathcal{U}$ denotes the number of elements of \mathcal{U} .

Biometric template protection algorithms We will give a explicit formulation of biometric template protection (BTP) algorithms as follows.

Definition 1 (BTP algorithms). *A biometric template protection (BTP) algorithm Π is a tuple of polynomial-time algorithms Gen, PIE, PIR, PIC, namely $\Pi = (\text{Gen}, \text{PIE}, \text{PIR}, \text{PIC})$. Let Gen is an algorithm which on input 1^k returns a finite set \mathcal{U} of biometric characteristics, the associated random variables X_u , $u \in \mathcal{U}$, over a semimetric space \mathcal{M} , and the public parameters \mathfrak{p} , where k is a security parameter. Let PIE be a randomized algorithm which on input $x \in \mathcal{M}$ returns a pair (π, α) of two data $\pi \in \mathcal{M}_{\text{PI}}$ and $\alpha \in \mathcal{M}_{\text{AD}}$, where \mathcal{M}_{AD} are finite sets. The algorithm PIE is called a pseudonymous identifier encoder. The first output π (resp. the second output α of PIE is called a pseudonymous identifier (PI) for enrollment (resp. auxiliary data (AD)) and is denoted by $\pi = \text{PIE}_1(x)$ (resp. $\alpha = \text{PIE}_2(x)$). The algorithm PIE can be regarded as a pair of two randomized algorithms PIE_1 and PIE_2 .*

In the enrollment phase, a biometric characteristic $u \in \mathcal{U}$ is submitted to the system, a feature element $x \in \mathcal{M}$ is generated according to the distribution X_u , PIE outputs (π, α) on input x , and π and α are stored in storages. Note that π and α are not necessarily stored together in the same storage.

Let PIR be a deterministic algorithm which, on input $\alpha \in \mathcal{M}_{\text{AD}}$ and $x' \in \mathcal{M}$, returns a data $\pi' \in \mathcal{M}'_{\text{PI}}$ for verification, where \mathcal{M}'_{PI} is a finite set. The data $\pi' = \text{PIR}(\alpha, x')$ is called a pseudonymous identifier for verification. Let PIC be a deterministic algorithm which, on input $\pi \in \mathcal{M}_{\text{PI}}$ and $\pi' \in \mathcal{M}'_{\text{PI}}$, returns either *match* or *non-match*. The algorithms PIR and PIC are called a pseudonymous identifier recorder and a pseudonymous identifier comparator, respectively.

In the verification phase, a biometric characteristic $u \in \mathcal{U}$ is freshly presented to the system, a new feature element $x' \in \mathcal{M}$ is generated according to X_u . The verification entity receives a PI π , an AD α and x' , computes $\pi' = \text{PIR}(\alpha, x')$, and outputs $\text{PIC}(\pi, \pi') \in \{\text{match}, \text{non-match}\}$.

Note that the terms, pseudonymous identifier (PI), auxiliary data (AD), are defined in ISO/IEC 24745 [6] (cf. [11]). A pseudonymous identifier (PI) is defined to be a set of data that represents an individual or data subject within a certain domain by means of a protected identity and is used as a reference for verification by means of a captured biometric sample and auxiliary data. It is desirable that the PI does not allow the retrieval of the enrolled biometric feature element and multiple “unlinkable” PIs can be derived from the same biometric characteristic. Auxiliary data (AD) is defined to be a set of data that can be required to reconstruct pseudonymous identifiers during verification. In some scheme, AD depends on the enrolled biometric feature element.

A pair (π, α) of PI and AD is called a *protected template (PT)* in [11] or a *renewable biometric reference* in [6]. In [11], in general, PTs are assumed to be public. However, most existing BTP algorithms require secrecy of PT. Because, in the real world, for some modalities (e.g. fingerprint, iris, face and so on) there are many public large databases, and therefore, the adversary can find a matching sample by entirely running such a database against a stolen PT. Therefore, in this paper, both PIs and ADs are assumed to be secret information. Each user’s PI and AD are separately stored in different storages, for example, in application to 2-factor authentication systems, every PI is stored together with each user’s ID in the database and each user’s AD is stored in the user’s smart card. We will discuss the recognition performance and the security performance when one of (or both) PI and AD is leaked. Simoens et al. [11] regard such a data separation as an additional property of BTP.

Definition 2 (2-factor BTP). *We will define a 2-factor BTP authentication algorithms in which a biometric characteristic is the first authentication factor. There are two possibilities from the viewpoint of data separation. A scheme which utilizes ADs as second factors and stores PIs for verification in the database is called a AD-2-factor BTP. Reversely, a scheme which utilizes PIs for verification as second factors and stores ADs in the database is called a PI-2-factor BTP.*

3 Recognition performance for BTP algorithms

In this section, we especially focus on recognition performance as technical performance of BTP algorithms Π . For the simplicity, we will fix a security parameter k . Therefore, a set \mathcal{U} of biometric characteristics, the associated random variables X_u , $u \in \mathcal{U}$, and the public parameters \mathbf{p} are fixed.

3.1 Accuracy

For any biometric template protection (BTP) algorithm $\Pi = (\text{PIE}, \text{PIR}, \text{PIC})$, the false non-match rate of Π , FNMR_{Π} , is the probability that a mated pair of PT and biometric sample are falsely

declared to be *non-match*, namely,

$$FNMR_{\Pi} = \mathbb{E}_{\substack{u \xleftarrow{\$} \mathcal{U} \\ x \leftarrow X_u \\ (\pi, \alpha) \leftarrow \text{PIE}(X_u)}} \Pr \left[\text{PIC}(\pi, \text{PIR}(\alpha, x)) = \text{non-match} \right]$$

Here, we will define recognition accuracy metrics for 2-factor BTPs, which are called *total performance* and naturally introduced from the notion, data separation, discussed by Simoens et al. [11, Section 4.4]. The *false match rate for total performance* of AD-2-factor BTP (resp. PI-2-factor BTP) Π , $FMR_{\Pi, \text{AD}}^{\text{TP}}$ (resp. $FMR_{\Pi, \text{PI}}^{\text{TP}}$), is the probability that a zero-effort impostor's presentation of his own biometric characteristic $u \in \mathcal{U}$ along with a 2nd factor $\alpha \in \mathcal{M}_{\text{AD}}$ (resp. $\pi \in \mathcal{M}_{\text{PI}}$) generated from u is falsely declared to match a non-mated reference data $\pi \in \mathcal{M}_{\text{PI}}$ (resp. $\alpha \in \mathcal{M}_{\text{AD}}$) generated from a biometric characteristic $v \in \mathcal{U} \setminus \{u\}$. The metrics $FMR_{\Pi, \text{AD}}^{\text{TP}}$ and $FMR_{\Pi, \text{PI}}^{\text{TP}}$ are respectively formulated by

$$FMR_{\Pi, \text{AD}}^{\text{TP}} = \mathbb{E}_{\substack{(u, v) \xleftarrow{\$} (\mathcal{U} \times \mathcal{U})^{\text{diff}} \\ x \leftarrow X_u \\ (\pi, \alpha) \leftarrow \text{PIE}(X_u) \\ (\pi', \alpha') \leftarrow \text{PIE}(X_v)}} \Pr \left[\text{PIC}(\pi', \text{PIR}(\alpha, x)) = \text{match} \right]$$

$$FMR_{\Pi, \text{PI}}^{\text{TP}} = \mathbb{E}_{\substack{(u, v) \xleftarrow{\$} (\mathcal{U} \times \mathcal{U})^{\text{diff}} \\ x \leftarrow X_u \\ (\pi, \alpha) \leftarrow \text{PIE}(X_u) \\ (\pi', \alpha') \leftarrow \text{PIE}(X_v)}} \Pr \left[\text{PIC}(\pi, \text{PIR}(\alpha', x)) = \text{match} \right] .$$

Nagar et al. [1] propose these metrics as the false accept rate with unknown transformation parameters, FAR_{UK} .

By measuring the above metrics, $FNMR_{\Pi}$, $FMR_{\Pi, \text{AD}}^{\text{TP}}$, and $FMR_{\Pi, \text{PI}}^{\text{TP}}$, we can totally evaluate the recognition performance of 2-factor BTPs. However, a 2-factor BTP can achieve a high recognition performance when the recognition accuracy contributed by one factor is high, even if the recognition accuracy contributed by the other factor is poor. Therefore, we need to evaluate the recognition accuracy achieved only by using one factor. In the following sections, Section 3.2 and 3.3, we will define metrics for such recognition accuracy.

3.2 Biometric Performance

In this section, we will define a metric for the recognition accuracy achieved only by the 1st factor, biometrics. The *false match rate for biometric performance* of Π , FMR_{Π}^{BP} , is the probability that a zero-effort impostor's presentation of his own biometric characteristic $u \in \mathcal{U}$ along with a correct 2nd factor is falsely declared to match a genuine reference data. Then the metric FMR_{Π}^{BP} is formulated by

$$FMR_{\Pi}^{\text{BP}} = \mathbb{E}_{\substack{(u, v) \xleftarrow{\$} (\mathcal{U} \times \mathcal{U})^{\text{diff}} \\ x \leftarrow X_u \\ (\pi, \alpha) \leftarrow \text{PIE}(X_v)}} \Pr [\text{PIC}(\pi, \text{PIR}(\alpha, x)) = \text{match}] .$$

Simoens et al. [11] discussed this metric as an ordinary recognition accuracy metric, the false match rate, because they mainly consider biometric-based single factor authentication systems

which stores PIs and ADs in the database. Moreover, Nagar et al. [1] propose this metric as *the false accept rate with known transformation parameters*, FAR_{KK} .

This metric can be regarded as a metric for security against impersonation when a user's 2nd factor is leaked. In the above notion, biometric performance, the adversary assumed to be very weak, namely he presents his own biometric characteristic along with obtained genuine user's 2nd factor. However, in order to strictly evaluate security against impersonation, we need to define a stronger attack model. We would discuss such a rigorous security in another paper in preparation.

3.3 Diversity

Diversity is the notion which ensures renewability for 2-factor BTPs. Namely, after a PT generated from $u \in \mathcal{U}$ is renewed, a presentation of u along with the old 2nd factor should not be declared to match the new reference data. Diversity is also the property that PTs should not allow cross-matching across databases in different authentication systems. (cf. [7, III], [1, Sect. 3.3], [11, Sect. 3.5]). We will define a metric for diversity as follows. The *false match rate for diversity* of BTP algorithm Π , FMR_{Π}^{Div} , is the probability that a presentation of a biometric characteristic $u \in \mathcal{U}$ along with a 2nd factor generated from u is falsely declared to match a new reference data freshly generated from the same u . The metrics FMR_{Π}^{Div} is formulated by

$$FMR_{\Pi}^{\text{Div}} = \mathbb{E}_{\substack{u \xleftarrow{\$} \mathcal{U} \\ x \leftarrow X_u \\ (\pi, \alpha) \leftarrow \text{PIE}(X_u) \\ (\pi', \alpha') \leftarrow \text{PIE}(X_u)}} \Pr \left[\text{PIC}(\pi, \text{PIR}(\alpha', x)) = \text{match} \right].$$

Nagar et al. [1] proposes this metric as the *cross match rate*, CMR . Here we consider the corresponding entropy $H = -\log FMR_{\Pi}^{\text{Div}}$. Then, it indicates that the distribution of PTs generated from a biometric characteristic are almost the same as the uniform distribution on H -bit binary strings, namely 2^H independent PTs can be generated from a biometric characteristic. Simoens et al. [11] propose the number of such “independent” PTs as a metric for diversity.

Diversity can be regarded as a metric for security against impersonation when a user's biometric characteristic is leaked. For example, in the real world, a fingerprint left on a glass is abused by a malicious user. However, in the above diversity notion, the adversary assumed to be very weak, namely he submits a 2nd factor randomly generated from the obtained biometric characteristic. By using the obtained biometric characteristic, a stronger adversary might be able to find a 2nd factor which makes PIC return *match* with extremely higher probability. We would discuss such a strict security notion in another paper in preparation.

4 Protection performance for BTP algorithms

4.1 Irreversibility

Suppose that the adversary obtains (a part of) a PT leaked from the database or from the user's storage devices. The adversary might be able to recover a feature element close to the original feature element from which the PT is generated. From the recovered feature element, he might create a physical spoof of the user's biometric characteristic and impersonate the user by presenting the fake biometric characteristic to the system. Irreversibility is a requirement that it should be

hard to recover an original feature element (or a neighborhood of it) from (a part of) a PT, which ensures the security in the case of leakage of PTs.

For each nonempty subset $\Lambda \neq \phi$ of the terms $\{PI, AD\}$ and any PT (π, α) , let $(\pi, \alpha)_\Lambda$ denote a subset of $\{\pi, \alpha\}$ defined by $(\pi, \alpha)_{\{PI, AD\}} = (\pi, \alpha)$, $(\pi, \alpha)_{\{PI\}} = \pi$, and $(\pi, \alpha)_{\{AD\}} = \alpha$. We call $(\pi, \alpha)_\Lambda$ a Λ -subset of (π, α) .

We will define a *irreversibility game (IRR Game)* between the challenger Ch and the adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, where \mathcal{A}_1 is a probabilistic polynomial-time (ppt) adversary which is given the algorithms and the parameters of Π and sends a state to \mathcal{A}_2 , and \mathcal{A}_2 is a ppt adversary who is given a Λ -subset of a PT generated from an feature element $x \in \mathcal{M}$ extracted from a randomly chosen biometric characteristic and attempts to guess (a neighborhood of) the original feature element x .

Recently, for most major modalities, there are many databases available to the public. Therefore, it is natural to assume that the adversary easily obtains a huge database of biometric samples. In this case, the adversary can performs an offline attack and successfully find a target feature element. In order to formulate such a practical situation, we will define an oracle from which the adversary can obtain feature elements corresponding to biometric characteristics submitted as queries. More precisely, let Samp be an oracle which, on input $u \in \mathcal{U}$, chooses $x \in \mathcal{M}$ according to X_u and returns x . We assume that the challenger and the adversary are allowed to make polynomial-time queries to Samp before he returns his guess.

For any subset $\phi \neq \Lambda \subset \{PI, AD\}$ and any real number $\tau \geq 0$, we define Λ - τ -authorized leakage game (Λ - AL_τ IRR Game) (resp. Λ -pseudo authorized leakage game (Λ -PAL IRR Game)) as follows.

Λ -AL $_\tau$ IRR Game (resp. **Λ -PAL IRR Game**)

Step 1. The challenger Ch inputs 1^k into Gen and Gen returns \mathcal{U} , X_u , $u \in \mathcal{U}$, and the parameters \mathbf{p} . The challenger Ch sends $(\mathbf{p}, \Lambda, \tau)$ (resp. (\mathbf{p}, Λ)) to the adversary \mathcal{A}_1 .

Step 2. The adversary \mathcal{A}_1 receives $(\mathbf{p}, \Lambda, \tau)$ (resp. (\mathbf{p}, Λ)) and sends a state s to \mathcal{A}_2 . The adversary \mathcal{A}_1 is allowed to make polynomial-time queries to Samp before he sends s to \mathcal{A}_2 .

Step 3. The challenger Ch chooses a biometric characteristic $u \in \mathcal{U}$ uniformly at random, submits u to the sampling oracle Samp, and gets a feature element $x \in \mathcal{M}$ as an answer from Samp. The challenger Ch inputs the feature element x into PIE, gets the output (π, α) , and sends $(\pi, \alpha)_\Lambda$ to the adversary \mathcal{A}_2 .

Step 4. The adversary \mathcal{A}_2 receives the state s and $(\pi, \alpha)_\Lambda$ from \mathcal{A}_1 and Ch, respectively, and returns $x' \in \mathcal{M}$. The adversary \mathcal{A}_2 is allowed to make polynomial-time queries to Samp before he returns his guess.

If $d(x, x') \leq \tau$ (resp. $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$), then the adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ wins.

Traditional biometric recognition algorithms, which do not use BTP algorithms, determines decision thresholds τ to minimize the false non-match rate $FNMR_{d \leq \tau}$ or the false match rate $FMR_{d \leq \tau}$ (cf. (1)). If the adversary obtains a Λ -subset of a PT and successfully recovers a feature element close to the original feature element, then he can impersonate the user in traditional authentication systems. Since some BTP algorithms might accept feature elements outside the τ -neighborhood of the original feature element, the adversary in Λ -PAL IRR Game might find a feature element x' such that $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$ but $d(x, x') > \tau$.

For any feature element $x \in \mathcal{M}$, the match rate of the feature element x with respect to $d \leq \tau$ (resp. the reverse match rate of the feature element x) $MR_{d \leq \tau}(x)$ (resp. $rMR_{\Pi}(x)$) is the probability that a feature element $x' \in \mathcal{M}$ (resp. a PT (π, α)) generated from a randomly chosen biometric characteristic $u \in \mathcal{U}$ satisfies $d(x, x') \leq \tau$ (resp. $\text{PIC}(\pi, \text{PIR}(\alpha, x)) = \text{match}$), which is formulated by

$$MR_{d \leq \tau}(x) = \mathbb{E}_{x' \leftarrow X(\mathcal{U})} \Pr [d(x, x') \leq \tau] \quad (2)$$

$$rMR_{\Pi}(x) = \mathbb{E}_{(\pi, \alpha) \leftarrow \text{PIE}(X(\mathcal{U}))} \Pr [\text{PIC}(\pi, \text{PIR}(\alpha, x)) = \text{match}] . \quad (3)$$

Put $m_{d \leq \tau} = \max_x MR_{d \leq \tau}(x)$ and $m_{\Pi} = \max_x rMR_{\Pi}(x)$. In $\Lambda\text{-AL}_{\tau}$ IRR Game (resp. $\Lambda\text{-PAL}$ IRR Game), the optimal strategy of an adversary \mathcal{A}' who is not given $(\pi, \alpha)_{\Lambda}$ is to return a feature element x satisfying $MR_{d \leq \tau}(x) = m_{d \leq \tau}$ (resp. $rMR_{\Pi}(x) = m_{\Pi}$) and then the success probability of the adversary \mathcal{A}' is equals to $m_{d \leq \tau}$ (resp. m_{Π}). Therefore, the advantage $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-AL}_{\tau} \text{ IRR}}$ (resp. $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-PAL IRR}}$) of the adversary \mathcal{A} is defined by

$$\begin{aligned} \text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-AL}_{\tau} \text{ IRR}} &= \Pr[\mathcal{A} \text{ in } \Lambda\text{-AL}_{\tau} \text{ IRR Game wins}] - m_{d \leq \tau} \\ \text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-PAL IRR}} &= \Pr[\mathcal{A} \text{ in } \Lambda\text{-PAL IRR Game wins}] - m_{\Pi} \end{aligned}$$

Definition 3 (Authorized-leakage irreversibility (cf. [11])). *We say that a BTP algorithm Π is $\varepsilon\text{-}\Lambda\text{-}\tau$ -authorized-leakage irreversible ($\varepsilon\text{-}\Lambda\text{-AL}_{\tau}$ IRR) if $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-AL}_{\tau} \text{ IRR}} < \varepsilon$ for any ppt adversary \mathcal{A} . In particular, we say that Π is $\varepsilon\text{-}\Lambda$ -full-leakage irreversible ($\varepsilon\text{-}\Lambda\text{-FL}$ IRR) if $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-AL}_0 \text{ IRR}} < \varepsilon$ for any ppt adversary \mathcal{A} .*

Definition 4 (Pseudo-authorized-leakage irreversibility (cf. [11])). *We say that a BTP algorithm Π is $\varepsilon\text{-}\Lambda$ -pseudo-authorized-leakage irreversible ($\varepsilon\text{-}\Lambda\text{-PAL}$ IRR) if $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-PAL IRR}} < \varepsilon$ for any ppt adversary \mathcal{A} .*

The above definitions immediately implies the following theorem. We omit the proof.

Theorem 1. *Fix any nonempty subset $\Lambda \subset \{\text{PI}, \text{AD}\}$ and any real numbers $\varepsilon > 0$ and $\tau \geq 0$. If a BTP algorithm Π is $\varepsilon\text{-}\Lambda\text{-AL}_{\tau}$ IRR, then Π is $(\varepsilon + m_{d \leq \tau} - m_{d \leq 0})\text{-}\Lambda\text{-FL}$ IRR.*

Moreover, assume that τ satisfies the condition that, for any $x \in \mathcal{M}$ and any PT (π, α) generated from x , $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$ if $d(x, x') \leq \tau$. If a BTP algorithm Π is $\varepsilon\text{-}\Lambda\text{-PAL}$ IRR, then Π is $(\varepsilon + m_{\text{PI}} - m_{d \leq \tau})\text{-}\Lambda\text{-AL}_{\tau}$ IRR.

Simoens et al. [11] also introduce the above metrics, FL IRR, AL IRR, and PAL IRR, as the difficulty of determining (a neighborhood of) the original feature element. Note that, in the attack model in [11] the adversary is given the whole PT. Here, we discuss unachievability of PAL IRR in the case when the adversary is given a whole PT, namely $\Lambda = \{\text{PI}, \text{AD}\}$. Actually, when the adversary obtains both the PI and the AD, he can find a target feature element with extremely high probability by making a certain amount of queries to Samp. We will more precisely discuss as follows.

For any PT $(\pi, \alpha) \in \mathcal{M}_{\text{PI}} \times \mathcal{M}_{\text{AD}}$, the match rate of the PT (π, α) , $MR_{\Pi}(\pi, \alpha)$, is the probability that a feature element $x' \in \mathcal{M}$ generated from a randomly chosen biometric characteristic $u \in \mathcal{U}$ satisfies $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$, which is formulated by

$$MR_{\Pi}(\pi, \alpha) = \mathbb{E}_{x' \leftarrow X(\mathcal{U})} \Pr [\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}] .$$

The $MR_{\Pi}(\pi, \alpha)$ can be regarded as a random variable over the distribution of (π, α) . Let MR_{Π} denote the average of $MR_{\Pi}(\pi, \alpha)$, namely

$$MR_{\Pi} = \mathbb{E}_{(\pi, \alpha) \leftarrow \text{PIE}(X(\mathcal{U}))} MR_{\Pi}(\pi, \alpha) .$$

Let σ is the standard deviation of the $MR_{\Pi}(\pi, \alpha)$. Then, from Chebyshev's inequality, we have

$$\Pr \left[MR_{\Pi}(\pi, \alpha) > MR_{\Pi} - \frac{\sigma}{\sqrt{\delta}} \right] \geq 1 - \delta \quad (4)$$

for any $\delta > 0$. For the simplicity, we assume that MR_{Π} and σ are constants independent of the security parameter k . Let C be the variation coefficient of $MR_{\Pi}(\pi, \alpha)$, namely $C = \frac{\sigma}{MR_{\Pi}}$. Assume that $C < 1$.

Theorem 2. *For all $\varepsilon < 1 - C^2 - m_{\Pi}$, there exists no ε -{PI, AD}-PAL IRR BTP algorithm.*

In general, more accurate BTP algorithms Π have smaller C . Therefore, Theorem 2 states that accurate BTP algorithms are unlikely to achieve sufficient irreversibility when both PI and AD are compromised.

We will prove Theorem 2 in Appendix A. We can also similarly prove unachievability of AL IRR when the adversary is given the whole PT under the assumptions slightly different from the case of PAL IRR. However, we omit a precise description of the statement and the proof in this e-print and will describe them in the full paper.

4.2 Unlinkability

For any nonempty subset $\Lambda \subset \{\text{PI}, \text{AD}\}$, we will define Λ -UNLINK Game between the challenger Ch and the adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$, where \mathcal{A} is given Λ -subsets of two PTs and attempts to guess whether the PTs are generated from the same biometric characteristics or not. In this game, Ch and \mathcal{A} are allowed to make polynomial-time queries to the sampling oracle Samp.

Λ -UNLINK Game

Step 1. The challenger Ch inputs 1^k into Gen and Gen returns $\mathcal{U}, X_u, u \in \mathcal{U}$, and the parameters \mathbf{p} . Ch sends (\mathbf{p}, Λ) to the adversary \mathcal{A}_1 .

Step 2. The adversary \mathcal{A}_1 receives (\mathbf{p}, Λ) , outputs three feature elements x, x_0, x_1 depending on a distribution selected by \mathcal{A}_1 , sends (x, x_0, x_1) to Ch, and sends a state s to \mathcal{A}_2 , where s contains (x, x_0, x_1) . The adversary \mathcal{A}_1 is allowed to make polynomial-time queries to Samp before he sends s to \mathcal{A}_2 .

Step 3. The challenger Ch flips the random coin $b \in \{0, 1\}$, inputs x, x_b into PIE, gets $PT = \text{PIE}(x)$ and $PT' = \text{PIE}(x_b)$, and sends $(PT)_{\Lambda}$ and $(PT')_{\Lambda}$ to the adversary \mathcal{A}_2 .

Step 4. The adversary \mathcal{A}_2 receives the state s and $(PT)_{\Lambda}$ and $(PT')_{\Lambda}$ from \mathcal{A}_1 and Ch, and returns $b' \in \{0, 1\}$ as a guess of b . The adversary \mathcal{A}_2 is allowed to make polynomial-time queries to Samp before he returns his guess.

If $b' = b$, then the adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ wins. The advantage $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-UNLINK}}$ of the adversary \mathcal{A} over the random guess is formulated by

$$\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-UNLINK}} = |2 \Pr[\mathcal{A} \text{ wins}] - 1| \quad (5)$$

Definition 5 (Unlinkability). *We say that a BTP algorithm Π is ε - Λ -unlinkable (ε - Λ -UNLINK), if $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-UNLINK}} < \varepsilon$ for any ppt adversary \mathcal{A} .*

Here, we will show unachievability of unlinkability when both PI and AD are compromised.

Theorem 3. *Assume that, for any $x \in \mathcal{M}$ and any $PT(\pi, \alpha)$ generated from x , $\text{PIC}(\pi, \text{PIR}(\alpha, x)) = \text{match}$. For any $\varepsilon \leq 1 - MR_{\Pi}$, there exists no ε -{PI, AD}-UNLINK BTP algorithm.*

In general, more accurate BTP algorithms Π have smaller MR_{Π} . Therefore, Theorem 3 states that accurate BTP algorithms are unlikely to achieve sufficient unlinkability when both PI and AD are compromised.

We will prove Theorem 3 in Appendix A.

Simoens et al. [11] define a metric for unlinkability by using the *false cross match rate (FCMR)* and the *false non-cross-match rate (FNCMR)*. They define an adversary $\mathcal{A}^{cc} = (\mathcal{A}_1^{cc}, \mathcal{A}_2^{cc})$, who is called the *cross-comparator*. In Λ -UNLINK Game, the adversary \mathcal{A}_1^{cc} chooses a pair $(u, v) \in (\mathcal{U} \times \mathcal{U})^{\text{diff}}$ of two different biometric characteristics, submits u to Samp independently twice, and receives x and x_0 respectively as the answers of two queries, moreover submits v to Samp, and receives x_1 as the answer, sends (x, x_0, x_1) to Ch, and sends a state s containing (x, x_0, x_1) to \mathcal{A}_2^{cc} . The adversary \mathcal{A}_2^{cc} receives the state containing (x, x_0, x_1) and $(PT_1)_{\Lambda}$ and $(PT_2)_{\Lambda}$ from \mathcal{A}_1^{cc} and Ch, respectively, and returns $b' \in \{0, 1\}$ as a guess of b .

The false cross match rate (*FCMR*) (resp. the false non-cross-match rate (*FNCMR*)) is the probability that, when $b = 1$ (resp. $b = 0$), the cross comparator \mathcal{A}^{cc} falsely guesses that $b' = 0$ (resp. $b' = 1$), which is formulated as follows:

$$\begin{aligned} FCMR_{\Pi, \mathcal{A}^{cc}}^{\Lambda\text{-UNLINK}} &= \mathbb{E}_{\substack{(u, v) \xleftarrow{\$} (\mathcal{U} \times \mathcal{U})^{\text{diff}} \\ (PT)_{\Lambda} \leftarrow \text{PIE}(X_u) \\ (PT')_{\Lambda} \leftarrow \text{PIE}(X_v)}} \Pr[\mathcal{A}^{cc} \text{ returns 0 in } \Lambda\text{-UNLINK Game}] \\ \left(\text{resp. } FNCMR_{\Pi, \mathcal{A}^{cc}}^{\Lambda\text{-UNLINK}} &= \mathbb{E}_{\substack{u \xleftarrow{\$} \mathcal{U} \\ (PT)_{\Lambda} \leftarrow \text{PIE}(X_u) \\ (PT')_{\Lambda} \leftarrow \text{PIE}(X_u)}} \Pr[\mathcal{A}^{cc} \text{ returns 1 in } \Lambda\text{-UNLINK Game}] \right). \end{aligned}$$

The advantage $\text{Adv}_{\Pi, \mathcal{A}^{cc}}^{\Lambda\text{-UNLINK}}$ of the cross comparator can be interpreted as follows:

$$\text{Adv}_{\Pi, \mathcal{A}^{cc}}^{\Lambda\text{-UNLINK}} = |1 - (FCMR_{\Pi, \mathcal{A}^{cc}}^{\Lambda\text{-UNLINK}} + FNCMR_{\Pi, \mathcal{A}^{cc}}^{\Lambda\text{-UNLINK}})|.$$

5 Relations among security notions

In this section, we will clarify relations among security notions, irreversibility and unlinkability, defined in the previous sections. We will prove that unlinkability is a stronger notion than

authorized-leakage irreversibility. Therefore, unlinkability gives more rigorous assurance on privacy than irreversibility. Before describing the precise statement, we will prepare some notations.

Let $P_\tau(x)$ be the probability that the τ -neighborhood of x' chosen according to the distribution $X(\mathcal{U})$ has non-empty intersection with $\mathcal{M}_\tau(x)$, namely

$$P_\tau(x) = \mathbb{E}_{x' \leftarrow X(\mathcal{U})} \Pr[\mathcal{M}_\tau(x) \cap \mathcal{M}_\tau(x') \neq \emptyset].$$

Note that, for any $\tau < \tau'$, $P_\tau(x) \leq P_{\tau'}(x)$. Put $p_\tau = \max_x P_\tau(x)$ and $q_\tau = \min_x P_\tau(x)$. Note that, for any $\tau < \tau'$, $p_\tau \leq p_{\tau'}$ and $q_\tau \leq q_{\tau'}$, and $q_0 \leq \frac{1}{\#\mathcal{M}} \leq p_0$ and the equality is attained if and only if $X(\mathcal{U})$ is a uniform distribution.

Theorem 4. *For any nonempty subset $\Lambda \subset \{\text{PI}, \text{AD}\}$, if a BTP algorithm Π is ε - Λ -UNLINK, then Π is $\frac{\varepsilon + (p_\tau - q_\tau)m_{d \leq \tau}}{1 - p_\tau}$ - Λ -AL $_\tau$ IRR for any $\tau \geq 0$.*

We will prove Theorem 4 in Appendix C.

From Theorem 1 and Theorem 4, we have the following figures, Figure 1 and Figure 2, which indicate relations among irreversibility and unlinkability when $\Lambda = \text{PI}$ and $\Lambda = \text{AD}$, respectively. The notation $A \longrightarrow B$ means that the notion A is stronger than the notion B . We avoid to show the figure in the case of $\Lambda = \{\text{PI}, \text{AD}\}$, because, as mentioned after Theorem 2 and Theorem 3, accurate BTP algorithms are unlikely to achieve sufficient irreversibility or unlinkability when both PI and AD are compromised.

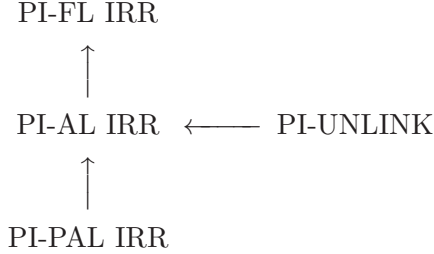


Figure 1: Relations among security notions when only PI is compromised.

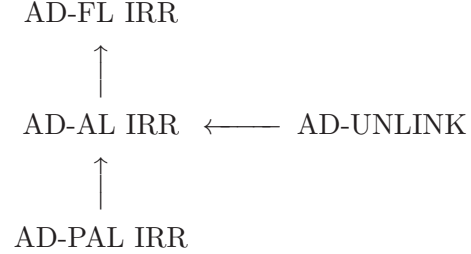


Figure 2: Relations among security notions when only AD is compromised.

References

- [1] A. Nagar, K. Nandakumar, and A. K. Jain. Biometric template transformation: A security analysis. In *Proc. SPIE, Electronic Imaging, Media Forensics and Security XII*, 2010.
- [2] A Arakala, J Jeffers, and K Horadam. Fuzzy extractors for minutiae-based fingerprint authentication. *Advances in Biometrics*, pages 760–769, 2007.
- [3] Ee-Chien Chang and Sujoy Roy. Robust extraction of secret bits from minutiae. In *ICB'07: Proceedings of the 2007 international conference on Advances in Biometrics*. Springer-Verlag, August 2007.

- [4] T Connie, A Teoh, M Goh, and D Ngo. PalmHashing: a novel approach for cancelable biometrics. *Information Processing Letters*, 93(1):1–5, 2005.
- [5] Y Dodis, L Reyzin, and A Smith. Fuzzy extractors: How to generate strong keys from biometrics and other noisy data. *Advances in cryptology-Eurocrypt 2004*, pages 523–540, 2004.
- [6] Information technology -Security techniques- Biometric information protection, 2011.
- [7] Anil K Jain, Karthik Nandakumar, and Abhishek Nagar. Biometric template security. *EURASIP Journal on Advances in Signal Processing*, 2008.
- [8] Andrew Teoh Beng Jin, David Ngo Chek Ling, and Alwyn Goh. Biohashing: two factor authentication featuring fingerprint data and tokenised random number. *Pattern recognition*, 37(11):2245–2255, November 2004.
- [9] A Juels and M Sudan. A fuzzy vault scheme. In *Information Theory, 2002. Proceedings. 2002 IEEE International Symposium on*, 2002.
- [10] Ari Juels and Martin Wattenberg. A fuzzy commitment scheme. In *CCS '99: Proceedings of the 6th ACM conference on Computer and communications security*. ACM Request Permissions, November 1999.
- [11] K. Simoens, B. Yang, X. Zhou, F. Beato, C. Busch, E. Newton, and B. Preneel. Criteria towards metrics for benchmakring template protection algoritms. In *Proc. of the 5th IAPR International Conference on Biometrics (ICB 2012)*, 2012.
- [12] A. Nagar, K Nandakumar, and A.K Jain. Biometric template transformation: a security analysis. *Proc. SPIE, Electronic Imaging, Media Forensics and Security XII*, 2010.
- [13] Nalini Ratha, Sharat Chikkerur, Jonathan Connell, and Ruud Bolle. Generating Cancelable Fingerprint Templates. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 29(4):561–572, 2007.
- [14] N.K Ratha, J.H Connell, and R.M Bolle. Enhancing security and privacy in biometrics-based authentication systems. *IBM Systems Journal*, 40(3):614–634, 2001.
- [15] Y Sutcu, Q Li, and N Memon. Protecting biometric templates with sketch: Theory and practice. *Information Forensics and Security, IEEE Transactions on*, 2(3):503–512, 2007.
- [16] A.B.J Teoh, A Goh, and D.C.L Ngo. Random Multispace Quantization as an Analytic Mechanism for BioHashing of Biometric and Random Identity Inputs. *Pattern Analysis and Machine Intelligence, IEEE Transactions on*, 28(12):1892–1901, 2006.
- [17] Andrew Beng Jin Teoh, Kar-Ann Toh, and Wai Kuan Yip. 2^N Discretisation of BioPhasor in cancellable biometrics , volume 4642 of *Lecture Notes in Computer Science*. Springer Berlin Heidelberg, Berlin, Heidelberg, 2007.
- [18] Wai Kuan Yip, Alwyn Goh, D Ngo, and A Teoh. Cryptographic Keys from Dynamic Hand-Signatures with Biometric Secrecy Preservation and Replaceability. In *Fourth IEEE Workshop on Automatic Identification Advanced Technologies (AutoID'05)*, pages 27–32. IEEE, 2005.

A Proof of Theorem 2

In this appendix, we will explicitly prove Theorem 2.

Proof of Theorem 2. Put $\Lambda = \{\text{PI}, \text{AD}\}$. We need to prove that, for any constant γ satisfying $C^2 < \gamma < 1$, there exists an adversary \mathcal{A} satisfying $\Pr[\mathcal{A} \text{ in } \Lambda\text{-PAL IRR Game wins}] > 1 - \gamma$. We will define an adversary \mathcal{A}_2 who obtains a PT (π, α) from the challenger Ch, makes polynomial-time queries to the sampling oracle Samp, and returns a guess $x' \in \mathcal{M}$.

Fix a constant δ satisfying $C^2 < \delta < \gamma$. Put $\mu = MR_{\Pi} - \frac{\sigma}{\sqrt{\delta}}$. Since $0 < \mu < 1$ and μ is a constant, there exists a constant number N_{δ} such that $(1 - \mu)^N < \frac{\gamma - \delta}{1 - \delta}$ for all $N \geq N_{\delta}$. The adversary \mathcal{A} repeats the following processes from Step 1 to Step 3 at most N_{δ} times.

Step 1. The adversary \mathcal{A}_2 chooses a biometric characteristic $v \in \mathcal{U}$ uniformly at random.

Step 2. The adversary \mathcal{A}_2 sends to the sampling oracle Samp and gets a feature element x' from Samp.

Step 3. The adversary \mathcal{A}_2 checks whether $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$ or *non-match*.

If $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$ in the Step 3 during the repetition of the above processes, then \mathcal{A}_2 finishes the processes and returns x' .

We say that a PT (π, α) is *good* if $MR_{\Pi}(\pi, \alpha) > \mu$. If the adversary \mathcal{A}_2 is given a good PT (π, α) , then the probability that \mathcal{A}_2 gets a feature element x' satisfying $\text{PIC}(\pi, \text{PIR}(\alpha, x')) = \text{match}$ during the N_{δ} -time repetition of the above steps is greater than or equal to $1 - (1 - \mu)^{N_{\delta}} > \frac{1 - \gamma}{1 - \delta}$. From (4), the probability that \mathcal{A}_2 is given a good PT (π, α) is greater than or equals to $1 - \delta$. Therefore, we have

$$\Pr[\mathcal{A} \text{ in } \Lambda\text{-PAL IRR Game wins}] > (1 - \delta) \times \frac{1 - \gamma}{1 - \delta} = 1 - \gamma.$$

Therefore, the result follows. \square

B Proof of Theorem 3

In this appendix, we will explicitly prove Theorem 3.

Proof of Theorem 3. It is sufficient to show that there exists an adversary \mathcal{A} in $\{\text{PI}, \text{AD}\}$ -UNLINK Game whose advantage is equal to $1 - MR_{\Pi}$. We define such an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ as follows.

The adversary \mathcal{A}_1 .

The adversary \mathcal{A}_1 receives $(\mathbf{p}, \{\text{PI}, \text{AD}\})$ from the challenger Ch, independently chooses three biometric characteristics $u, u_0, u_1 \in \mathcal{U}$ uniformly at random, makes queries u, u_0, u_1 to Samp, gets three feature elements x, x_0, x_1 from Samp, respectively, sends (x, x_0, x_1) to Ch, and sends a state $s' = ((x, x_0, x_1), \mathbf{p}, \{\text{PI}, \text{AD}\})$ to \mathcal{A}_2 .

The adversary \mathcal{A}_2 .

The adversary \mathcal{A}_2 receives the state $s' = ((x, x_0, x_1), \mathbf{p}, \{\text{PI}, \text{AD}\})$ and $PT = (\pi, \alpha)$ and $PT' =$

(π', α') from \mathcal{A}_1 and Ch , respectively. When $\text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{non-match}$, \mathcal{A}_2 puts $b' = 0$. When $\text{PIC}(\pi', \text{PIR}(\alpha', x_0)) = \text{non-match}$, \mathcal{A}_2 puts $b' = 1$. When $\text{PIC}(\pi', \text{PIR}(\alpha', x_0)) = \text{match}$ and $\text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{match}$, \mathcal{A}_2 chooses b' from $\{0, 1\}$ uniformly at random. Finally \mathcal{A}_2 returns b' .

From the assumption in the statement of Theorem 3, if $(\pi', \alpha') = \text{PIE}(x_0)$ (resp. $(\pi', \alpha') = \text{PIE}(x_1)$), then $\text{PIC}(\pi', \text{PIR}(\alpha', x_0)) = \text{match}$ (resp. $\text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{match}$). Therefore, when $b = 0$, there are the following two cases in which \mathcal{A} correctly returns $b' = 0$.

Case 1. $\text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{non-match}$

Case 2. $\text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{match}$ and $b' = 0$ is chosen from $\{0, 1\}$ with probability $\frac{1}{2}$.

Therefore, the probability that, when $b = 0$, the adversary \mathcal{A} correctly returns $b' = 0$ is estimated as follows:

$$\begin{aligned} \Pr[\mathcal{A} \text{ returns } b' = 0 \mid b = 0] &= \mathbb{E}_{\substack{(x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ (\pi', \alpha') \leftarrow \text{PIE}(x_0)}}} \Pr [\text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{non-match}] \\ &\quad + \mathbb{E}_{\substack{(x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ (\pi', \alpha') \leftarrow \text{PIE}(x_0)}}} \Pr \left[\begin{array}{l} \text{PIC}(\pi', \text{PIR}(\alpha', x_1)) = \text{match} \\ b' = 0 \xleftarrow{\$} \{0, 1\} \end{array} \right] \\ &= (1 - MR_{\Pi}) + \frac{1}{2} MR_{\Pi} = 1 - \frac{1}{2} MR_{\Pi} . \end{aligned}$$

The success probability of \mathcal{A} when $b = 1$ is similarly estimated as follows:

$$\Pr[\mathcal{A} \text{ returns } b' = 1 \mid b = 1] = 1 - \frac{1}{2} MR_{\Pi} .$$

Hence, we have

$$\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-UNLINK}} = |2 \Pr[\mathcal{A} \text{ in } \Lambda\text{-UNLINK Game wins}] - 1| = |2(1 - \frac{1}{2} MR_{\Pi}) - 1| = 1 - MR_{\Pi}$$

Therefore the result follows. \square

C Proof of Theorem 4

In this appendix, we will explicitly prove Theorem 4.

Proof of Theorem 4. Put $\varepsilon' = \frac{\varepsilon + (p_{\tau} - q_{\tau})m_{d \leq \tau}}{1 - p_{\tau}}$. It is sufficient to show that if there exists an adversary \mathcal{A} in $\Lambda\text{-AL}_{\tau}$ IRR Game whose advantage is greater than or equal to ε' , then there exists an adversary \mathcal{B} in $\Lambda\text{-UNLINK}$ Game whose advantage is greater than or equal to ε . Suppose that there exists an adversary $\mathcal{A} = (\mathcal{A}_1, \mathcal{A}_2)$ satisfying $\text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-AL}_{\tau} \text{ IRR}} \geq \varepsilon'$ in $\Lambda\text{-AL}_{\tau}$ IRR Game. We define an adversary $\mathcal{B} = (\mathcal{B}_1, \mathcal{B}_2)$ in $\Lambda\text{-UNLINK}$ Game as follows.

The adversary \mathcal{B}_1 .

The adversary \mathcal{B}_1 receives (\mathbf{p}, Λ) from the challenger Ch, inputs (\mathbf{p}, Λ) into the adversary \mathcal{A}_1 , and obtains a state s as an output of $\mathcal{A}_1(\mathbf{p}, \Lambda)$. Then, \mathcal{B}_1 independently chooses three biometric characteristics $u, u_0, u_1 \in \mathcal{U}$ uniformly at random, makes queries u, u_0, u_1 to Samp, gets three feature elements x, x_0, x_1 from Samp, respectively, sends (x, x_0, x_1) to Ch, and sends a state $s' = ((x, x_0, x_1), s)$ to \mathcal{B}_2 .

The adversary \mathcal{B}_2 .

The adversary \mathcal{B}_2 receives the state $s' = ((x, x_0, x_1), s)$ and $(PT)_\Lambda$ and $(PT')_\Lambda$ from \mathcal{B}_1 and Ch, respectively. When $\mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi$, \mathcal{B}_2 inputs s and $(PT')_\Lambda$ into \mathcal{A}_2 and obtains a feature element x' as an output of $\mathcal{A}_2(s, (PT')_\Lambda)$. If $d(x_0, x') \leq \tau$, then $b' = 0$, if $d(x_1, x') \leq \tau$, then $b' = 1$, otherwise b' is chosen from $\{0, 1\}$ uniformly at random. When $\mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) \neq \phi$, b' is also chosen from $\{0, 1\}$ uniformly at random. Finally \mathcal{B}_2 returns b' .

When $b = 0$, there are the following three cases in which the adversary \mathcal{B} correctly returns $b' = 0$.

Case 1. $\mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi$ and \mathcal{A}_2 guesses a feature element x' satisfying $d(x_0, x') \leq \tau$.

Case 2. $\mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi$, \mathcal{A}_2 guesses a feature element x' satisfying $d(x_0, x') > \tau$ and $d(x_1, x') > \tau$, and $b' = 0$ is chosen from $\{0, 1\}$ with probability $\frac{1}{2}$.

Case 3. $\mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) \neq \phi$ and $b' = 0$ is chosen from $\{0, 1\}$ with probability $\frac{1}{2}$.

Therefore, the probability that, when $b = 0$, the adversary \mathcal{B} correctly returns $b' = 0$ is expanded as follows:

$$\begin{aligned}
& \Pr[\mathcal{B} \text{ returns } b' = 0 \mid b = 0] \\
&= \mathop{\mathbb{E}}_{\substack{(x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0)}} \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_0, x') \leq \tau \end{array} \right] \\
&+ \mathop{\mathbb{E}}_{\substack{(x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0)}} \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_0, x') > \tau, d(x_1, x') > \tau \\ b' = 0 \xleftarrow{\$} \{0, 1\} \end{array} \right] \\
&+ \mathop{\mathbb{E}}_{\substack{(x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0)}} \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) \neq \phi \\ b' = 0 \xleftarrow{\$} \{0, 1\} \end{array} \right].
\end{aligned}$$

Since $\Pr[E_1 \cap (\neg E_2 \cap \neg E_3)] \geq \Pr[E_1] - (\Pr[E_1 \cap E_2] + \Pr[E_1 \cap E_3])$ for any events E_1, E_2 , and E_3 ,

the second term is estimated as follows:

$$\begin{aligned}
& \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_0, x') > \tau, d(x_1, x') > \tau \\ b' = 0 \stackrel{\$}{\leftarrow} \{0, 1\} \end{array} \right] \\
& \geq \frac{1}{2} \left(\Pr[\mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi] - \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_0, x') \leq \tau \end{array} \right] \right. \\
& \quad \left. - \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_1, x') \leq \tau \end{array} \right] \right) .
\end{aligned}$$

Therefore, we have

$$\begin{aligned}
& \Pr[\mathcal{B} \text{ returns } b' = 0 \mid b = 0] \\
& \geq \frac{1}{2} \left(\begin{array}{l} \text{E} \\ (x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0) \end{array} \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_0, x') \leq \tau \end{array} \right] \right. \\
& \quad \left. - \begin{array}{l} \text{E} \\ (x_0, x_1) \leftarrow X(\mathcal{U}) \times X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0) \end{array} \Pr \left[\begin{array}{l} \mathcal{M}_\tau(x_0) \cap \mathcal{M}_\tau(x_1) = \phi \\ \mathcal{A}_2((PT')_\Lambda) = x', d(x_1, x') \leq \tau \end{array} \right] + 1 \right) .
\end{aligned}$$

By the definitions of p_τ and q_τ , we have

$$\begin{aligned}
& \Pr[\mathcal{B} \text{ returns } b' = 0 \mid b = 0] \\
& \geq \frac{1}{2} \left((1 - p_\tau) \begin{array}{l} \text{E} \\ x_0 \leftarrow X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0) \end{array} \Pr[\mathcal{A}_2((PT')_\Lambda) = x', d(x_0, x') \leq \tau] \right. \\
& \quad \left. - (1 - q_\tau) \begin{array}{l} \text{E} \\ x_1 \leftarrow X(\mathcal{U}) \\ PT' \leftarrow \text{PIE}(x_0) \end{array} \Pr[\mathcal{A}_2((PT')_\Lambda) = x', d(x_1, x') \leq \tau] + 1 \right) .
\end{aligned}$$

Since \mathcal{A}_2 is only given independent information from $x_1 \in \mathcal{M}$, the probability that \mathcal{A}_2 guess a feature element x' contained in the τ -neighborhood to x_1 is at most $m_{d \leq \tau}$. Consequently, we have

$$\Pr[\mathcal{B} \text{ returns } b' = 0 \mid b = 0] \geq \frac{1}{2} \left((1 - p_\tau) \Pr[\mathcal{A} \text{ in } \Lambda\text{-AL}_\tau \text{ IRR Game wins}] - (1 - q_\tau)m_{d \leq \tau} + 1 \right) .$$

We can similarly estimate the success probability of \mathcal{B} when $b = 1$ as follows:

$$\Pr[\mathcal{B} \text{ returns } b' = 1 \mid b = 1] \geq \frac{1}{2} \left((1 - p_\tau) \Pr[\mathcal{A} \text{ in } \Lambda\text{-AL}_\tau \text{ IRR Game wins}] - (1 - q_\tau)m_{d \leq \tau} + 1 \right) .$$

Finally, the advantage of the adversary \mathcal{B} is calculated as follows:

$$\begin{aligned}
\text{Adv}_{\Pi, \mathcal{B}}^{\Lambda\text{-UNLINK}} &= |2 \Pr[\mathcal{B} \text{ in } \Lambda\text{-UNLINK Game wins}] - 1| \\
&\geq |(1 - p_\tau) \Pr[\mathcal{A} \text{ in } \Lambda\text{-AL}_\tau \text{ IRR Game wins}] - (1 - q_\tau)m_{d \leq \tau}| \\
&= |(1 - p_\tau) \text{Adv}_{\Pi, \mathcal{A}}^{\Lambda\text{-AL}_\tau \text{ IRR}} - (p_\tau - q_\tau)m_{d \leq \tau}| \\
&\geq |(1 - p_\tau)\varepsilon' - (p_\tau - q_\tau)m_{d \leq \tau}| = \varepsilon .
\end{aligned}$$

Therefore the result follows. □